ERRATUM TO "THE CENTRALIZER OF A NILPOTENT SECTION"

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The paper (McNinch 2008) contains an error concerning the smoothness of certain group schemes. Specifically, the statements (2.3.1) and (2.3.2) of *loc. cit.* – which give conditions for the smoothness of the stabilizer of a section – are incorrect. The conditions given by these statements fail to guarantee the *flatness* of the stabilizer group scheme, which is necessary for the smoothness..

Let $\mathcal{A}$ be a complete discrete valuation ring with fractions K and residue field k, and let $\mathcal{G}$ be a split reductive group scheme over $\mathcal{A}$. The results of the preceding paragraph were applied in *loc. cit.* §5.2 to conclude the smoothness over $\mathcal{A}$ of the centralizer $C = C_{\mathcal{G}}(X)$ of a nilpotent section $X \in \mathrm{Lie}(\mathcal{G}) = \mathrm{Lie}(\mathcal{G})(\mathcal{A})$ provided that C_{K} is a smooth group scheme over K, that C_{k} is a smooth group scheme over k, and that $\dim C_{\mathrm{K}} = \dim C_{\mathrm{k}}$. Because of the error noted above, that conclusion is not supported by the given arguments.

As a consequence, the proof of (McNinch 2008, Theorem B) is incorrect. That Theorem asserts that the component group of a nilpotent centralizer is – in a suitable sense – independent of "good" characteristic. The arguments of (McNinch 2008, §7) all depend on the smoothness of the *full* centralizer $C = C_{\mathcal{G}}(X)$, which has not been confirmed. I am not aware of any cases where the concusion is known to be false. See (Booher 2016) where the smoothness is verified in some special cases.

The given proof of (McNinch 2008, Theorem A) is also incorrect, but here the situation is repairable. Theorem A of the paper asserts that the root datum of the reductive quotient of the centralizer of a nilpotent element is in a suitable sense independent of "good" characteristic.

To repair the argument, one can use the following result, whose proof was communicated to me by Brian Conrad. More details can be found in (McNinch 2016).

Theorem 1 (Brian Conrad)**.** *Let $\mathcal{H}$ be a group scheme of finite type over $\mathcal{A}$ for which the fibers $\mathcal{H}_{\mathrm{K}}$ and $\mathcal{H}_{\mathrm{k}}$ are each smooth of the same dimension. Then there is a locally closed subgroup scheme $\mathcal{M} \subset \mathcal{H}$ such that:*

(a) $\mathcal{M}$ is smooth, affine, and of finite type over $\mathcal{A}$,
(b) $\mathcal{M}_{\mathrm{K}} = (\mathcal{H}_{\mathrm{K}})^0$ and $\mathcal{M}_{\mathrm{k}} = (\mathcal{H}_{\mathrm{k}})^0$.

To prove (McNinch 2008, Theorem A), one argues as in *loc. cit.*, but in the arguments one must replace the identity component C^0 of the centralizer $C = C_{\mathcal{G}}(X)$ – which is not known to be smooth over $\mathcal{A}$ – with the locally closed subgroup smooth scheme $\mathcal{M} \subset C$ whose existence is guaranteed by the preceding theorem.

For more details, see (McNinch 2016, §8), where we give an updated proof of Theorem A.

REFERENCES

Booher, Jeremy (2016). "Geometric Deformations of Orthogonal and Symplectic Galois Representations". Ph.D. Thesis; Stanford University.

McNinch, George (2008). "The centralizer of a nilpotent section". In: *Nagoya Math. J.* 190, pp. 129–181.

– (2016). *On the nilpotent orbits of a reductive group over a local field*. preprint. URL: http://math.tufts.edu/faculty/gmcninch/manuscripts.html.

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