

ERRATUM TO “THE CENTRALIZER OF A NILPOTENT SECTION”

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The paper (McNinch 2008) contains an error concerning the smoothness of certain group schemes. Specifically, the statements (2.3.1) and (2.3.2) of *loc. cit.* – which give conditions for the smoothness of the stabilizer of a section – are incorrect. The conditions given by these statements fail to guarantee the *flatness* of the stabilizer group scheme, which is necessary for the smoothness..

Let \mathcal{A} be a complete discrete valuation ring with fractions K and residue field k , and let \mathcal{G} be a split reductive group scheme over \mathcal{A} . The results of the preceding paragraph were applied in *loc. cit.* §5.2 to conclude the smoothness over \mathcal{A} of the centralizer $C = C_{\mathcal{G}}(X)$ of a nilpotent section $X \in \text{Lie}(\mathcal{G}) = \text{Lie}(\mathcal{G})(\mathcal{A})$ provided that C_K is a smooth group scheme over K , that C_k is a smooth group scheme over k , and that $\dim C_K = \dim C_k$. Because of the error noted above, that conclusion is not supported by the given arguments.

As a consequence, the proof of (McNinch 2008, Theorem B) is incorrect. That Theorem asserts that the component group of a nilpotent centralizer is – in a suitable sense – independent of “good” characteristic. The arguments of (McNinch 2008, §7) all depend on the smoothness of the *full* centralizer $C = C_{\mathcal{G}}(X)$, which has not been confirmed. Note that I am not aware of any cases where the conclusion is known to be false; see (Booher 2016) where the smoothness is verified in some special cases.

The given proof of (McNinch 2008, Theorem A) also invoked the smoothness of C , but here the situation is repairable. Theorem A of the paper asserts that the root datum of the reductive quotient of the centralizer of a nilpotent element is in a suitable sense independent of “good” characteristic.

To repair the argument, one can use (McNinch 2021, Prop. 3.5.1). This result gives an open subgroup scheme \mathcal{M} of the centralizer which is connected and reductive – and in particular, smooth – over \mathcal{A} , and for which \mathcal{M}_K and \mathcal{M}_k are, respectively, Levi factors of the connected centralizers C_K^0 and C_k^0 .

One can now repeat the proof of Theorem A given in (McNinch 2008, §5.8), using \mathcal{M} rather than the full centralizer C .

REFERENCES

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